CALCULUS AB SECTION I, Part A

Time — 55 minutes

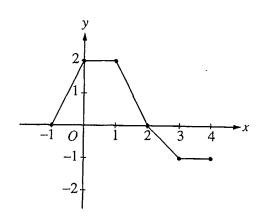
Number of questions — 28

#### A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

- 1. What is the x-coordinate of the point of inflection on the graph of  $y = \frac{1}{3}x^3 + 5x^2 + 24$ ?
  - (A) 5
- (B) 0
- (C)  $-\frac{10}{3}$
- (E) 10



- 2. The graph of a piecewise-linear function f, for  $-1 \le x \le 4$ , is shown above. What is the value of  $\int_{-1}^4 f(x) \ dx$ ?
  - (A) 1
- (B) 2.5
- (C) 4
- (D) 5.5
- (E) 8

- $3. \qquad \int_1^2 \frac{1}{x^2} \, dx =$ 
  - (A)  $-\frac{1}{2}$
- (B)  $\frac{7}{24}$
- (C)  $\frac{1}{2}$
- (D) 1
- (E) 2 ln 2

4. If f is continuous for  $a \le x \le b$  and differentiable for a < x < b, which of the following could be

- (A)  $f'(c) = \frac{f(b) f(a)}{b a}$  for some c such that a < c < b.
- (B) f'(c) = 0 for some c such that a < c < b.
- (C) f has a minimum value on  $a \le x \le b$ .
- (D) f has a maximum value on  $a \le x \le b$ .
- (E)  $\int_{-\infty}^{\infty} f(x) dx$  exists.

 $\int_0^{\infty} \sin t \, dt =$ 

- (A)  $\sin x$
- (B)  $-\cos x$
- (C)  $\cos x$
- (D)  $\cos x 1$

- 6. If  $x^2 + xy = 10$ , then when x = 2,  $\frac{dy}{dx} = -1$
- (A)  $-\frac{7}{2}$
- (B) -2
- (C)  $\frac{2}{7}$

$$7. \qquad \int_{1}^{e} \left( \frac{x^2 - 1}{x} \right) dx =$$

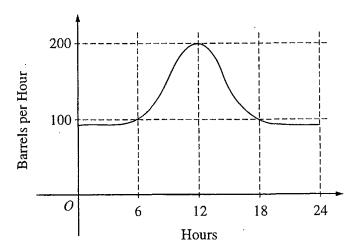
- (A)  $e \frac{1}{e}$  (B)  $e^2 e$  (C)  $\frac{e^2}{2} e + \frac{1}{2}$  (D)  $e^2 2$  (E)  $\frac{e^2}{2} \frac{3}{2}$

8. Let f and g be differentiable functions with the following properties:

- (i) g(x) > 0 for all x
- (ii) f(0) = 1

If h(x) = f(x)g(x) and h'(x) = f(x)g'(x), then f(x) =

- (A) f'(x)
- (B) g(x)
- (C)  $e^x$
- (D) 0
- (E) 1



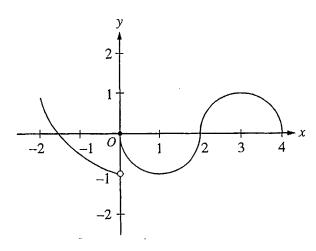
- 9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?
  - (A) 500
- (B) 600
- (C) 2,400
- (D) 3,000
- (E) 4,800

- 10. What is the instantaneous rate of change at x = 2 of the function f given by  $f(x) = \frac{x^2 2}{x 1}$ ?
  - (A) -2
- (B)  $\frac{1}{6}$
- (C)  $\frac{1}{2}$  (D) 2

- 11. If f is a linear function and 0 < a < b, then  $\int_{a}^{b} f''(x) dx =$ 
  - (A) 0
- (C)  $\frac{ab}{2}$
- (D) b a

12. If  $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2\\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then  $\lim f(x)$  is  $x\rightarrow 2$ 

- (A) ln 2
- (B) ln 8
- (C) ln 16
- (D) 4
- (E) nonexistent



- 13. The graph of the function f shown in the figure above has a vertical tangent at the point (2, 0) and horizontal tangents at the points (1, -1) and (3, 1). For what values of x, -2 < x < 4, is f not differentiable?
  - (A) 0 only
- (B) 0 and 2 only
- (C) 1 and 3 only
- (D) 0, 1, and 3 only
- (E) 0, 1, 2, and 3

- 14. A particle moves along the x-axis so that its position at time t is given by  $x(t) = t^2 6t + 5$ . For what value of t is the velocity of the particle zero?
  - (A) 1

(B) 2

(C) 3

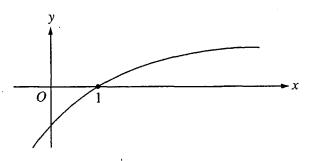
(D) 4

(E) 5

- 15. If  $F(x) = \int_0^x \sqrt{t^3 + 1} dt$ , then F'(2) =
  - (A) -3
- (B) -2
- (C) 2
- (D) 3
- (E) 18

16. If  $f(x) = \sin(e^{-x})$ , then f'(x) =

- (A)  $-\cos(e^{-x})$
- (B)  $\cos(e^{-x}) + e^{-x}$
- (C)  $\cos(e^{-x}) e^{-x}$
- (D)  $e^{-x} \cos(e^{-x})$
- (E)  $-e^{-x}\cos(e^{-x})$



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

- (A) f(1) < f'(1) < f''(1)
- (B) f(1) < f''(1) < f'(1)
- (C) f'(1) < f(1) < f''(1)
- (D) f''(1) < f(1) < f'(1)
- (E) f''(1) < f'(1) < f(1)

- 18. An equation of the line tangent to the graph of  $y = x + \cos x$  at the point (0, 1) is
  - (A) y = 2x + 1
- (B)  $\dot{y} = x + 1$
- (C) y = x
- (D) y = x 1
- (E) y = 0

- 19. If  $f''(x) = x(x+1)(x-2)^2$ , then the graph of f has inflection points when x =
  - (A) -1 only
- (B) 2 only
- (C) -1 and 0 only
- (D) -1 and 2 only
- (E) -1, 0, and 2 only

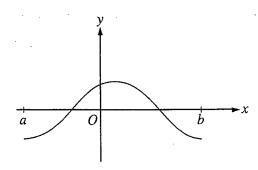
- 20. What are all values of k for which  $\int_{-3}^{k} x^2 dx = 0$ ?
  - (A) -3
- (B) 0
- (C) 3
- (D) -3 and 3
- (E) -3, 0, and 3

- 21. If  $\frac{dy}{dt} = ky$  and k is a nonzero constant, then y could be
  - (A)  $2e^{kty}$

- (B)  $2e^{kt}$  (C)  $e^{kt} + 3$  (D) kty + 5 (E)  $\frac{1}{2}ky^2 + \frac{1}{2}$

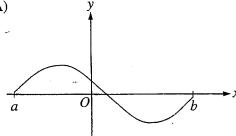
22. The function f is given by  $f(x) = x^4 + x^2 - 2$ . On which of the following intervals is f increasing?

- $(A) \cdot \left(-\frac{1}{\sqrt{2}}, \infty\right)$
- (B)  $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
- (C) (0, ∞)
- (D)  $(-\infty, 0)$
- (E)  $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

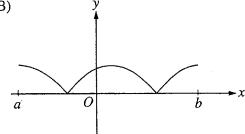


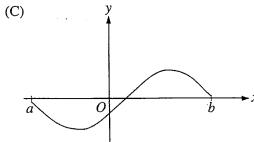
23. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

(A)

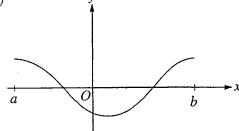


(B)

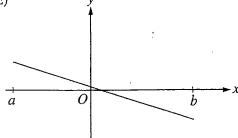




(D)



(E)



- 24. The maximum acceleration attained on the interval  $0 \le t \le 3$  by the particle whose velocity is given by  $v(t) = t^3 3t^2 + 12t + 4$  is
  - (A) 9
- (B) 12
- (C) 14
- (D) 21
- (E) 40

- 25. What is the area of the region between the graphs of  $y = x^2$  and y = -x from x = 0 to x = 2?
  - (A)  $\frac{2}{3}$
- (B)  $\frac{8}{3}$
- (C) 4
- (D)  $\frac{14}{3}$
- (E)  $\frac{16}{3}$

х	0 1		2	
f(x)	1	k	2	

- 26. The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation  $f(x) = \frac{1}{2}$  must have at least two solutions in the interval [0, 2] if k =
  - (A) 0

- (B)  $\frac{1}{2}$
- (C) 1

- (D) 2
- (E) 3

- 27. What is the average value of  $y = x^2 \sqrt{x^3 + 1}$  on the interval [0, 2]?
  - (A)  $\frac{26}{9}$
- (B)  $\frac{52}{9}$  (C)  $\frac{26}{3}$
- (D)  $\frac{52}{3}$
- (E) 24

28. If  $f(x) = \tan(2x)$ , then  $f'\left(\frac{\pi}{6}\right) =$ 

(A)  $\sqrt{3}$ 

(B)  $2\sqrt{3}$ 

(C) 4

(D)  $4\sqrt{3}$ 

(E) 8

#### END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS PART ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB SECTION I, Part B Time — 50 minutes Number of questions — 17

#### A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.

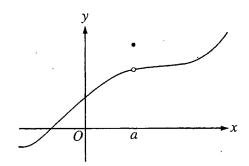
Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76-92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

#### In this test:

- The exact numerical value of the correct answer does not always appear among the choices given. (1) When this happens, select from among the choices the number that best approximates the exact numerical value.
- (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

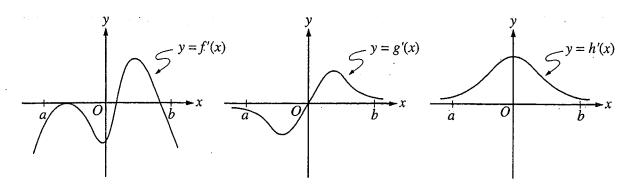


- 76. The graph of a function f is shown above. Which of the following statements about f is false?
  - (A) f is continuous at x = a.
  - (B) f has a relative maximum at x = a.
  - (C) x = a is in the domain of f.
  - (D)  $\lim_{x\to a^+} f(x)$  is equal to  $\lim_{x\to a^-} f(x)$ .
  - (E)  $\lim_{x \to a} f(x)$  exists.

- 77. Let f be the function given by  $f(x) = 3e^{2x}$  and let g be the function given by  $g(x) = 6x^3$ . At what value of x do the graphs of f and g have parallel tangent lines?
  - (A) -0.701
  - (B) -0.567
  - (C) -0.391
  - (D) -0.302
  - (E) -0.258

- 78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?
  - (A)  $-(0.2)\pi C$
  - (B) -(0.1)C

  - (D)  $(0.1)^2 C$
  - (E)  $(0.1)^2 \pi C$



- 79. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval a < x < b?
  - (A) f only
  - (B) g only
  - (C) h only
  - (D) f and g only (E) f, g, and h

- 80. The first derivative of the function f is given by  $f'(x) = \frac{\cos^2 x}{x} \frac{1}{5}$ . How many critical values does f have on the open interval (0, 10)?
  - (A) One
  - (B) Three
  - (C) Four
  - (D) Five
  - (E) Seven

81. Let f be the function given by f(x) = |x|. Which of the following statements about f are true?

- I. f is continuous at x = 0.
- II. f is differentiable at x = 0.
- III. f has an absolute minimum at x = 0.
- (A) I only
- (B) II only
- (C) III only
- (D) I and III only
- (E) II and III only

82. If f is a continuous function and if F'(x) = f(x) for all real numbers x, then  $\int_{1}^{3} f(2x)dx =$ 

- (A) 2F(3) 2F(1)
- (B)  $\frac{1}{2}F(3) \frac{1}{2}F(1)$
- (C) 2F(6) 2F(2)
- (D) F(6) F(2)
  - (E)  $\frac{1}{2}F(6) \frac{1}{2}F(2)$

- 83. If  $a \neq 0$ , then  $\lim_{x \to a} \frac{x^2 a^2}{x^4 a^4}$  is
  - (A)  $\frac{1}{a^2}$
- (B)  $\frac{1}{2a^2}$  (C)  $\frac{1}{6a^2}$
- (D) 0
- (E) nonexistent

- 84. Population y grows according to the equation  $\frac{dy}{dt} = ky$ , where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is
  - (A) 0.069
- (B) 0.200
- (C) 0.301
- (D) 3.322
- (E) 5.000

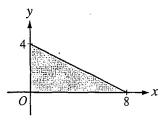
x	2	5	7	8	
f(x)	10	30	40	20	

85. The function f is continuous on the closed interval [2, 8] and has values that are given in the table

above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of

$$\int_{2}^{8} f(x) \, dx ?$$

- (A) 110
- (B) 130
- (C) 160
- (D) 190
- (E) 210



- 86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line x + 2y = 8, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?
  - (A) 12.566
- (B) 14.661
- (C) 16.755
- (D) 67.021
- (E) 134.041

- 87. Which of the following is an equation of the line tangent to the graph of  $f(x) = x^4 + 2x^2$  at the point where f'(x) = 1?
  - (A) y = 8x 5
  - (B) y = x + 7
  - (C) y = x + 0.763
  - (D) y = x 0.122
  - (E) y = x 2.146

- 88. Let F(x) be an antiderivative of  $\frac{(\ln x)^3}{x}$ . If F(1) = 0, then F(9) =
  - (A) 0.048
- (B) 0.144
- (C) 5.827
- (D) 23.308
- (E) 1,640.250

89. If g is a differentiable function such that g(x) < 0 for all real numbers x and if  $f'(x) = (x^2 - 4)g(x)$ , which of the following is true?

- (A) f has a relative maximum at x = -2 and a relative minimum at x = 2.
- (B) f has a relative minimum at x = -2 and a relative maximum at x = 2.
- (C) f has relative minima at x = -2 and at x = 2.
- (D) f has relative maxima at x = -2 and at x = 2.
- (E) It cannot be determined if f has any relative extrema.

90. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

- (A) A is always increasing.
- (B) A is always decreasing.
- (C) A is decreasing only when b < h.
- (D) A is decreasing only when b > h.
- (E) A remains constant.

- 91. Let f be a function that is differentiable on the open interval (1, 10). If f(2) = -5, f(5) = 5, and f(9) = -5, which of the following must be true?
  - I. f has at least 2 zeros.
  - II. The graph of f has at least one horizontal tangent. III. For some c, 2 < c < 5, f(c) = 3.

  - (A) None
  - (B) I only
  - (C) I and II only
  - (D) I and III only
  - (E) I, II and III

- 92. If  $0 \le k < \frac{\pi}{2}$  and the area under the curve  $y = \cos x$  from x = k to  $x = \frac{\pi}{2}$  is 0.1, then  $k = \frac{\pi}{2}$ 
  - (A) 1.471
- (B) 1.414
- (C) 1.277
- (D) 1.120
- (E) 0.436

#### END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY. DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET AND HAVE WRITTEN AND GRIDDED YOUR AP NUMBER IN THE APPROPRIATE SECTION OF YOUR ANSWER SHEET.

AFTER TIME HAS BEEN CALLED, ANSWER QUESTIONS 93-96.

- 93. Which graphing calculator did you use during the examination?
  - (A) Casio 6300, Casio 7000, Casio 7300, Casio 7400, or Casio 7700
  - (B) Texas Instruments TI-80 or TI-81
  - (C) Casio 9700, Casio 9800, Casio 9850, Sharp 9200, Sharp 9300, Texas Instruments TI-82, Texas Instruments TI-83, Texas Instruments TI-85, or Texas Instruments TI-86
  - (D) Hewlett-Packard HP-48 series or HP-38G
  - (E) Some other calculator
- 94. During your Calculus AB course, which of the following best describes your calculator use?
  - (A) I used my own graphing calculator.
  - (B) I used a graphing calculator furnished by my school, both in class and at home.
  - (C) I used a graphing calculator furnished by my school only in class.
  - (D) I used a graphing calculator furnished by my school mostly in class, but occasionally at home.
  - (E) I did not use a graphing calculator.
- 95. During your Calculus AB course, which of the following describes approximately how often a graphing calculator was used by you or your teacher in classroom learning activities?
  - (A) Almost every class
  - (B) About three-quarters of the classes
  - (C) About one-half of the classes
  - (D) About one-quarter of the classes
  - (E) Seldom or never
- 96. During your Calculus AB course, which of the following describes approximately how often you were allowed to use a graphing calculator on tests?
  - (A) Almost all of the time
  - (B) About three-quarters of the time
  - (C) About one-half of the time
  - (D) About one-quarter of the time
  - (E) Seldom or never

### Gapan

# Answers to the 1998 AP Calculus AB and Calculus BC Examinations

- Section I: Multiple Choice
  - Blank Answer Sheet
- Section II: Free Response
  - B Student Preparation for the Exams
  - Free-Response Questions, Scoring Guidelines, and Sample Student Responses with Commentary
  - Section II, Calculus AB
  - Section II, Calculus BC

## Section 1 Answer Key and Percent Answering Correctly Calculus AB

Percent Correct by Grade

1/0	Answei	5			- 2	1	Contest
<b>建图1</b> 53	D	96%	92%	83%	70%	41%	78%
<b>15.2</b> 2 5	В	75%	55%	37%	24%	19%	43%
5.3	C	94%	85%	75%	60%	38%	71%
$4^{r_1}$	В	82%	62%	47%	36%	-31%	52%
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10 (c)	A	94%	86%	75%	53%	-21%	68%
全包膏	E	85%	59%	38%	22%	13%	, 43%
<b>学图</b>	E	77%	53%	43%	37%	31%	48%
5 9	Ď	90%	77%	65%	55%	45%	67%
510点	D	91%	83%	7.4%	62%	62%	74%
MARK .	Α	76%	50%	36%	27%	17%	42%
12	Ε	91%	71%	48%	29%	. 20%	52%
13	В	72%	55%	41%	31%	20%	46%
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16	E	94%	87%	74%	54%	29%	69%
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18	B	96%	86%	72%	54%	35%	71%
19	С	55%	36%	23%	17%	10%	28%
20	Α	88%	83%	71%	62%	39%	69%
21	В	59%	34%	25%	20%	23%	31%
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26_	Α	80%	51%	31%	21%	16%	41%
_27	A	72%	50%	33%	21%	13%	40%
28	Ε'	86%	71%	51%	32%	9%	56%
76	A	79%	65%	59%	56%	49%	61%
2.77	C	96%	82%	63%	41%	25%	63%
78	- В	87%	66%	42%	22%	9%	46%
79	Ā	84%	66%	46%	27%	11%	47%
80	В	72%	57%	46%	34%	26%	48%
81	Ď.	93%	82%	73%	66%	51%	- 74%
82	E	47%	28%	19%	15%	14%	24%
TANALI							200
83	В	81%	60%	40%	28%	19%	45%
84	A	82%	57%	38%	21%	11%	42%
E 85	Ç	80%	56%	42%	29%	- 21%	46%
86 (	C	33%	15%	12%	17%	23%	19%
87	D	80%	50%	32%	20%	17%	40%
88	C	89%	71%	54%	38%	24%	55%
89.	В	76%	48%	31%	19%	17%	38%
- 90	D	70%	45%	30%	20%	15%	36%
91	E	80%	64%	49%	34%	18%	51%
92	D	97%	91%	80%	62%	30%	76%

#### Section I: Multiple Choice

Listed below are the correct answers to the multiplechoice questions and the percentage of AP candidates who answered each question correctly. A copy of the blank answer sheet appears on the following pages for reference.

Section 1 Answer Key and Percent Answering Correctly
Calculus BC

Calculus DC							
ltem No	Correct Answer	5	Percent 4	Correct 3	by Grade 2	1	Total Percent Correct
1 2 3 4 5	C A D A A	91% 98% 80% 83% 84%	81% 96% 63% 65% 64%	79% 92% 50% 49% 51%	67% 85% 30% 32% 35%	55% 72% 26% 19% 20%	80% 92% 58% 61% 60%
6 7 8 9	E E B D E	93% 89% 78% 91% 90%	89% 74% 60% 83% 78%	84% 61% 41% 76% 72%	69% 41% 28% 67% 64%	50% 31% 13% 58% 39%	83% 68% 55% 80% 75%
11 12 13 14 15	A E B E B	76% 90% 74% 86% 89%	57% 77% 66% 73% 80%	44% 64% 58% 62% 71%	34% 49% 45% 43% 63%	18% 27% 29% 31% 41%	55% 72% 62% 68% 76%
16 - 17 - 18 - 19 - 20	C D B D	55% 77% 57% 56% 79%	40% 59% 36% 34% 56%	29% 41% 23% 27% 39%	25% 24% 21% 17% 32%	15% 11% 13% 13% 9%	37% 54% 35% 37% 53%
21 22 23 24 25	C A E C C	89% 83% 53% 60% 68%	78% 69% 28% 38% 44%	66% 59% 21% 24% 33%	46% 56% 14% 17% 16%	38% 47% 8% 12% 17%	73% 68% 31% 38% 45%
26 27 28 76 77	E D C D E	32% 49% 72% 86% 94%	16% 29% 47% 64% 90%	10% 27% 46% 47% 85%	12% 16% 33% 34% 83%	13% 17% 25% 22% 70%	20% 35% 56% 60% 87%
78' 79 80 81 82	B A B B	88% 57% 96% 43% 72%	76% 47% 92% 21% 44%	68% 33% 93% 16% 38%	41% 33% 86% 10% 28%	28% 22% 69% 7% 16%	69% 45% 91% 25% 48%
83 84 85 86 87	C B C C	44% 63% 78% 50% 79%	24% 38% 58% 26% 56%	13% 27% 45% 19% 42%	12% 20% 38% 11% 35%	11% 12% 24% 17% 21%	27% 40% 58% 32% 57%
-88 -89 -190 -21 -91	C A A E D	93% 74% 46% 37% 86%	83% 55% 31% 25% 66%	66% 46% 26% 17% 46%	48% - 37% - 20% - 17% - 31%	34% 27% 9% 14%	75% 56% 32% 26% 65%